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Theoretical analysis of magnetoresistance commensurability oscillations of 2D electrons in antidot lattice in the presence of an in-plane magnetic field

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Abstract

The classical dynamics of the two-dimensional electron gas in antidot lattice is studied in the presence of an in-plane magnetic field. We find theoretically that the distortion of the Fermi contour induced by the in-plane magnetic field affects severely the position and amplitude of the commensurability magnetoresistance oscillations. Calculations of the Poincare sections show a strong variation of the two-dimensional electron gas dynamics.

Keywords: Antidots lattice; Chaotic dynamics; Magnetotransport; 2-D electron gas

1. Introduction

Progress in lithography has made it possible to create devices with size smaller than ballistic mean free path. Such devices, called electron billiards, allow to study the ballistic regime, which is mainly governed by the shape of the boundary of the sample. For certain irregular shapes of the billiards the motion of electrons becomes completely unpredictable after first few collisions, which means that chaos develops in the phase space. Within the last few years it has been shown that the two-dimensional array of antidots is a model system which allows to study the chaotic classical dynamics in condensed matter physics [1]. The general interest in these phenomena is a quantum manifestation of the classical chaos which could create the link between the so called quantum and classical chaos. In a periodical array of antidots the commensurability peaks in magnetoresistance oscillations have been already explained from the classical and the quantum point of view [2,3]. Recently it has been predicted that the Fermi contour of the two-dimensional electron gas can be distorted by the in plane magnetic field [4]. In this work the sensitivity of the electron chaotic dynamics in an antidots lattice to the Fermi contour distortion is studied theoretically.

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2. Theory

The dynamics of a two-dimensional electron gas (2DEG) in a periodic antidot array and perpendicular magnetic field is described by the Hamiltonian

$$H = 1/2m(p_x + eBy/2)^2 + 1/2m(p_y - eBx/2)^2 + U(x, y) \tag{1}$$

The antidot lattice can be described by the following model potential

$$U(x, y) = U_0\{\cos(2\pi x/d)\cos(2\pi y/d)\}^n \tag{2}$$

where n controls the steepness of the antidots, d is the lattice periodicity. We consider a smooth potential with $n = 6$ to $n = 20$ for weak electrostatic modulation, and a step potential with $n > 30$ for strong modulation. In order to solve the Hamiltonian equation of motion (1) and to find electron trajectories for different values of magnetic field, we use dimensionless variables

$$\tilde{x} = \frac{x}{d}, \quad \tilde{y} = \frac{y}{d}, \quad \tilde{t} = \frac{t}{\tau_0}, \quad \tilde{H} = \frac{H}{E_F}, \quad \tilde{U} = \frac{U}{E_F} \tag{3}$$

where d is the period of the antidot lattice, and E_F is the Fermi energy. As units to scale time and the magnetic field strength we use [5]

$$\tau_0 = \left[\frac{md^2}{2E_F}\right]^{1/2} \tag{4}$$

$$B_0 = \frac{2(2mE_F)^{1/2}}{ed} \tag{5}$$

$B = B_0$ is the field strength for a cyclotron radius equal to half the distance between two adjacent antidots. In this way the equation for the electron motion reads

$$\tilde{H} = \left[\tilde{p}_x + \frac{B}{B_0}\tilde{y}\right]^2 + \left[\tilde{p}_y - \frac{B}{B_0}\tilde{x}\right]^2 + \tilde{U}(\tilde{x}, \tilde{y}) \tag{6}$$

To understand the dynamic of the nonlinear systems it is very useful to investigate the motion in phase space by means of Poincare sections. By using the analytical model described above we calculate Poincare sections (P_x, x) at $y = 0$. Fig. 1 shows four Poincare sections for different values of magnetic field. The ballistic orbit of electron is bent with the magnetic field and focused into a collecting point. Therefore the geometric resonance in transport phenomena is observed at a magnetic field, where an integer multiple of the cyclotron diameter coincides with the periodicity of antidot lattice. Fig. 1d shows a Poincare section for magnetic field strength which corresponds to a cyclotron diameter $2R_c$ ($R_c = h/eB(2\pi n_s)^{1/2}$, n_s is electron density) equal to d. We see that for these conditions well defined quasiperiodic orbits appear (islands), characterized by a cycle in the phase space, surrounded by the chaotic component. These quasiperiodic orbits are responsible for the magneto-resistance peaks. These peaks were interpreted on the basis of pinned classical orbits in a 'pinball model' [1]. In accordance with this model electrons are pinned in orbits around (or between) groups of antidots for a long time and, therefore, do not contribute to the conductivity. Other unperturbed cyclotron orbits are represented by closed loops in the island. An alternative interpretation for the fundamental peaks occurring at magnetic field was proposed in [2,3], where the peak is due to an

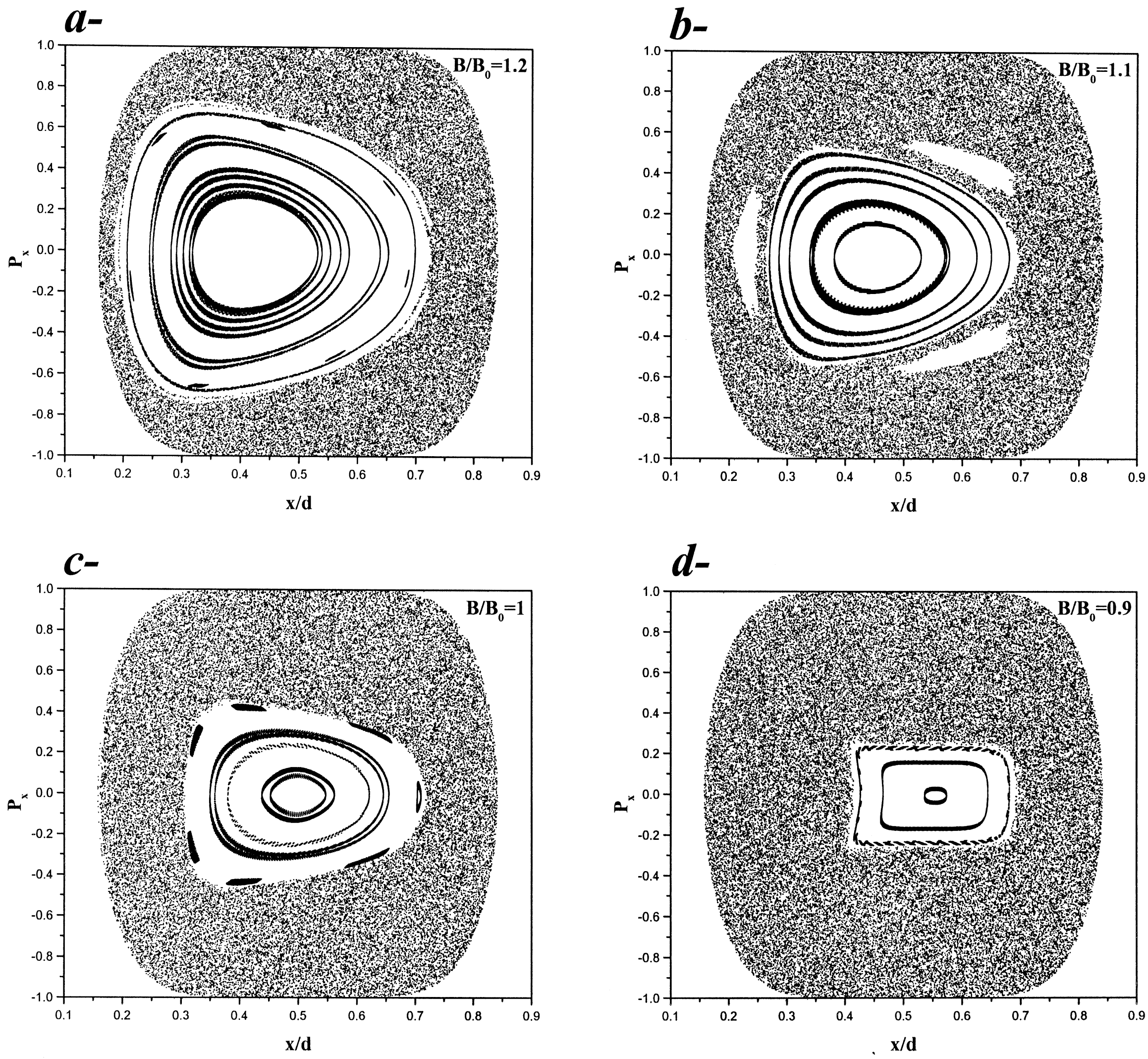

Fig. 1. Poincare sections at $x = 0$ for different values of magnetic field around the resonant condition $B/B_0 = 1$. The traced and solid lines represent periodic and quasiperiodic orbits due to 'pinned' trajectories. These islands of regular motion are surrounded by a chaotic sea.

enhancement of the diffusion coefficient produced by 'running' orbits, which skip regularly from an antidot to the other antidot. These orbits appear in Poincare sections at lower magnetic field. For low value of magnetic field the region in phase space covered by these quasiperiodic orbits (pinned) is comparable with the size of chaotic orbits, and as B is decreased, only the chaotic part remains.

Let us consider now the effect of the in-plane magnetic field, $B_{\|}$ on a 2DEG confined in the z direction, assuming for the sake of simplicity the harmonic confining potential with characteristic energy $h\Omega$. In this case the electron Hamiltonian has the following form

$$H = 1/2m(p_x/m + \omega_{\parallel}z)^2 + p_y^2/2m + p_z^2/2m + m\Omega^2 z^2/2m \tag{7}$$

where $w_{\parallel} = eB_{\parallel}/m$ is a cyclotron frequency. The energy dispersion of the electron subband in this case is given as follows

$$E(k) = h^2 k_y^2/2m + h^2 k_x^2/2m^* \tag{8}$$

where $m^* = m(1 + w_{\parallel}^2/\Omega^2)$ represents the increase of the effective mass induced by the in-plane magnetic field $B_{\parallel}$. From Eq. (4) we can see that the Fermi contour has an egg-like shape, instead of the circle isotropic shape in the absence of in-plane magnetic field [4]. Such anisotropy of the Fermi contour induces distortion of the electron trajectories and, consequently electron dynamics in antidot lattice. To test the sensitivity of the electron dynamics to the Fermi contour distortion we compute the Poincare section considering the anisotropy of the effective mass. We substitute into the equation of motion the value of new effective mass calculated in [6]. Fig. 2 shows three Poincare sections for the resonance condition $B/B_0 = 1$. The ratio of the antidot period to the antidot diameter at the Fermi energy (d/a) was held constant and equal to 4.14. Fig. 2a shows the poincare section in the presence of perpendicular magnetic field. Fig. 2b and c show Poincare sections for the same initial conditions of Fig. 2a, but now in the presence of two different values of in-plane magnetic field components. We see that the Poincare section for the geometric resonance condition is dramatically changed when a parallel magnetic field is applied. This is because the stability of the regular orbits is sensitive to the geometry of the experiment. Anisotropy of the effective mass induces anisotropy of the cyclotron diameter and changes resonance conditions in the x direction. It means that pinned cyclotron orbits with different initial conditions are absent. Therefore, the chaotic dynamic in the antidot lattice can be used to study the small change in the shape of the electron Fermi contour under the parallel magnetic field, pressure and other external factors.

Computations of the Poincare sections can be used to give quantitative information on the stability of the periodic orbits. By calculating the area of the phase space the fraction of periodic orbits can in principle obtained. The value of magnetic field when this area is a maximum gives information about the position of the maxima in the magnetoresistance.

For a further understanding of the chaotic dynamics in an antidot lattice in the presence of in-plane magnetic fields and of the role of the different type of trajectories, the numerical calculations of the diffusion coefficient and velocity correlation function, also were made. The magnetoresistance was obtained from the longitudinal conductivity using classical linear response theory [7]. The conductivity tensor is given by

$$\sigma_{ij} = \frac{m^* e^2}{\pi\hbar^2} \int_0^\infty \langle v_i(t) v_j(0) \rangle_\Gamma e^{-t/\tau}\, \mathrm{d}t \tag{9}$$

where $\langle v_i(t) v_j(0) \rangle_\Gamma$ is the velocity self-correlation function averaged over phase space, and $f(t) = e^{-t/\tau}$ represents the amount of electrons which do not collide within a time interval $[0, t]$. τ is the average time between two successive collisions.

Magnetoresistance at low magnetic field is obtained from the expression

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \tag{10}$$

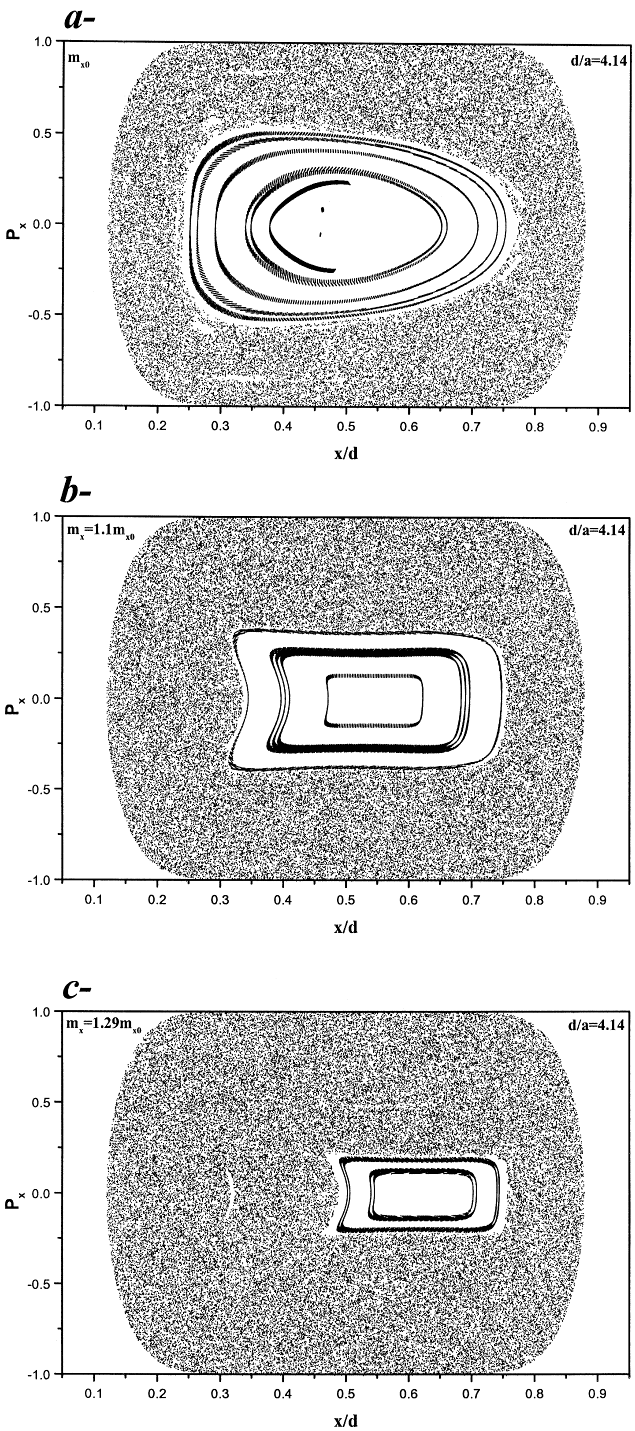

Fig. 2. (a) Poincare section for $d/a = 4.14$ in perpendicular field. (b), (c) Poincare sections for the same initial conditions as in (a), now in the presence of increasing in-plane magnetic field component.

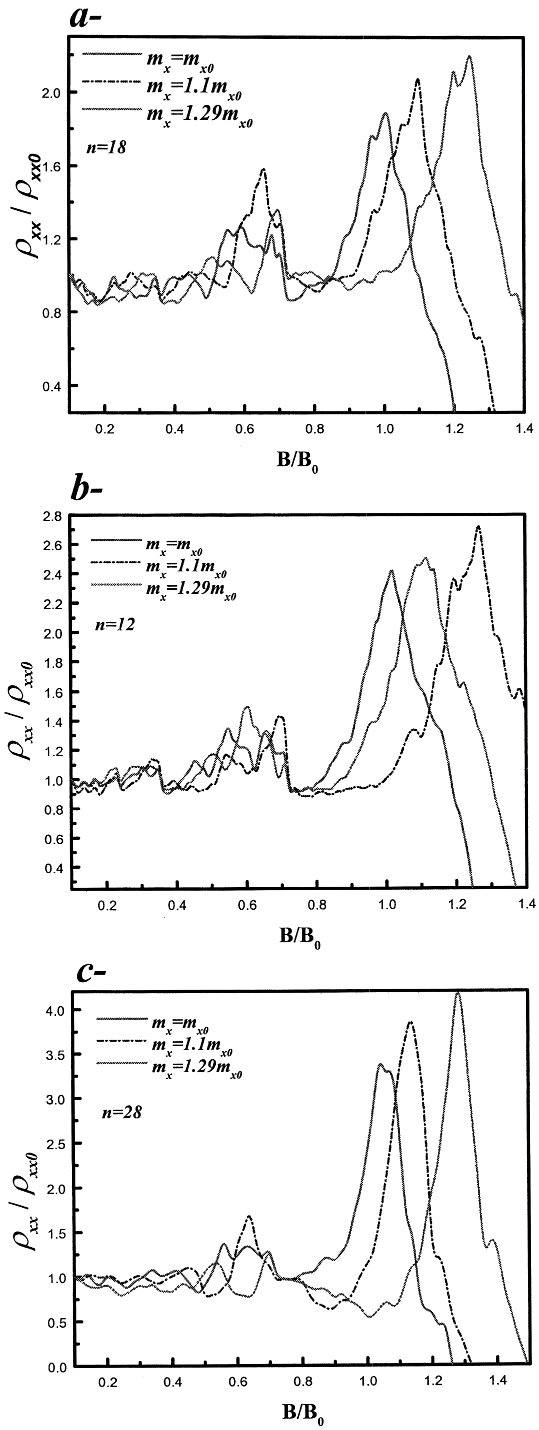

Fig. 3. Calculated in-plane magnetic field dependence of the commensurability oscillations. In each figure m_{x_0} represents the effective mass in the x direction and in perpendicular magnetic field. m_x indicates the variation of m_{x_0} in tilted fields.

Fig. 3a, b and c show the calculated in-plane magnetic field dependence of the commensurability peaks. For these calculations we used $n = 12$, $n = 18$ and $n = 28$, respectively. The distortion of the Fermi contour can be traduced as an increment of the effective mass in the x direction, this effect changes the electron trajectories from circular to elliptical, strongly modifying the stability and dynamics of the electron gas. The calculated results for magnetoresistance oscillations show a variation on the position, amplitude and shape of the commensurability peaks. The modification of the cyclotronlike trajectories from circular to elliptical produces a change on the geometric resonance condition, in this way a different number of electrons can be enclosed by 'pinned orbits', this suggests a shifting and distortion of the peaks' positions. The increasing of chaotic trajectories also suggests an enhancement of the diffusion coefficient and also of the longitudinal resistance.

From the comparison between the Poincare sections calculated for $B_{\perp}$ and those calculated with an in-plane component we find that closed loops, which are due to the formation of the pinned orbits with different initial conditions, are destroyed in the presence of an in-plane magnetic field. Therefore we can assume that the dynamics of these orbits can be responsible for the negative magnetoresistance in an antidot lattice in magnetic fields when $2R_c < d$.

3. Conclusion

In conclusion we demonstrate theoretically that the chaotic dynamics of electrons in an antidot lattice is very sensitive to the distortion of the Fermi contour. Therefore, the magnetoresistance of the 2DEG in the antidot lattice can be used to study such phenomena as the topological Lifschitz transition and small changes in the shape of the electron Fermi contour under pressure.

Acknowledgements

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